Aspects of ICT in Mathematical Activity: Tool and Media

Morten Misfeldt

Abstract—In this article two different approaches to artefacts that support mathematical activity and leaning are investigated; the instrumental approach, which concerns the way that artefacts are made into personal instruments, and the semiotic approach, which concerns the way semiotic representations influence mathematical activity. The motivation for applying these two views of the use of ICT in mathematical activities, is twofold: firstly, computational technology which is used to support mathematical work always involves semiotic representations. Secondly, such representations are used in mathematical activities as a tool that has to be learned and mastered, and which significantly affects solution techniques.

Index Terms— ***E-learning, GeoGebra, Information technology, Instrumental approach, Mathematics ,Semiotics, Writing***

1. INTRODUCTION

WHEN ICT is used in mathematical activities, two main uses of the technology are sought realized. The first is that ICT can work as a medium for mathematical representations, and thus support both communication and the cognitive mathematical processes that have been shown to rely on external representation (Galison, 2003, DiSessa, 2000, Duval 2006, Winsløw 2003). The second is that ICT works as a tool, and changes various problem situations in mathematics education by allowing easy graphing, algebraic and numerical computations, and visualisations (Dreyfuss 1994, Drijvers & Gravemeijer, 2005, Mariotti 2002, Trouche 2005).

In this article, I address these two uses and the fact that they are often treated as different topics in different theoretical constructs.

In the article, I give a brief and general background for the importance of artefacts for mathematical activities; describe why the semiotic aspects of such artefacts and the basic semiotic units (such as sign and medium) are important in relation to mathematical activities. Furthermore, I describe two theories from mathematics education: (1) Duval's theory of the role of semiotic representations in mathematics education and (2) Truche's instrumental approach. The reason for comparing these two theories is that semiotic representations can act as cognitive tools, and the tools that we use in mathematical activity have semiotic properties which both theoretical frameworks try to address; a comparison will show the strength and weaknesses of each framework. I use these two frames to discuss some examples of ICT use in mathematical activity, researchers' mathematical writing, e-learning at university level and the use of dynamic geometry software.

2. INTERNAL AND EXTERNAL ASPECTS OF MATHEMATICAL ACTIVITY

Mathematical activity often relies on an interplay between internal processes, only perceivable by the individual, and external actions, that are also observable by others.

In a Piagetian psychological framework the relation between internal and external processes is expressed as a definition of cognition as an adaptive function developed from – and tested against – empirical reality through actions (Glasersfeld 1995, p. 59). This view of cognition allows for theories of mathematical learning that emphasise relational thinking (Skemp, 1971) and abstraction as a result of particular manipulation with mathematical objects (Dubinsky, 1992). In that sense, mathematical reasoning is grounded in activities external to the mind.

In a sociocultural tradition, the concept 'mediated activity' designates how tools and signs influence and support human activity (Vygotskij, 1978, p. 51). By expanding on Actor Network Theory, Shaffer and Clinton (2006) introduces the concept 'toolforthought' and claim: "In this ontology, then, there are no tools without thinking, and there is no thinking without tools. There are only toolforthoughts, which represent the reciprocal relation between tools and thoughts that exists in both" (p. 291). The concept 'toolforthought' essentially remove the distinction between tools and thoughts, and consider human cognition a matter of working with 'toolforthoughts'.

Semiotic artefacts such as text, diagrams, tables, ect. are important in much knowledge work, not at least in mathematics. Signs and representations on paper can be used to support thinking.

The study of mathematical activity is connected

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to the study of how people interpret and manipulate their environment. In relation to mathematical activity, I consider two important types of 'toolforthoughts' and two theories to describe their use in mathematical activities. The two types are semiotic representations (signs, drawings, diagrams ect.), and the use of computational tools (calculators, Computer Algebra Systems and Dynamic Geometry).

2.1 The instrumental approach

The instrumental approach to the use of technology in mathematics education is developed partly from the discipline 'cognitive ergonomics' (Verillion 1995), and partly from the theory of 'conceptual fields' (Vergnaud 1996).

Verillion and Rabardel works in a sociocultural tradition, end from the assumption that artefacts mediate and shape human agency, the scope of their work is human-computer interaction. Vergnaud is concerned with science and mathematics learning. He is building on the work on Piaget and especially the concept 'scheme': "A scheme is the invariant organization of behaviour (action) for a certain class of situations." (Vergnaud p. 222).

This definition is important, because the instrumental approach is used to examine the process in which the introduction of new technology changes schemes.

The instrumental approach is used to examine how people who use artefacts to address a problem create personal instruments and instrumented techniques to address these problems. Luc Trouche defines an artefact as "a material or abstract object, aiming to sustain human activity" (Trouche, 2005, p. 144). An instrument is what is what the subject builds from the artefact. The process of building an instrument from an artefact is referred to as 'instrumental genesis' and consists of two processes, 'instrumentation' and 'instrumentalisation'. Instrumentation is directed by the subject, towards the artefact. This process includes several phases: discovery, selection, personalisation and transformation. While the first involves getting to know the tool, the latter tends to be a matter of mastering the tool and applying it to one's own very specific needs. Instrumentalisation is directed by the artefact towards the subject. It is the process in which the subject adapts to the new opportunities and constraints the tools offer. In the instrumentation process, the tool shapes the behaviour of the subject confronted with a specific type of tasks.

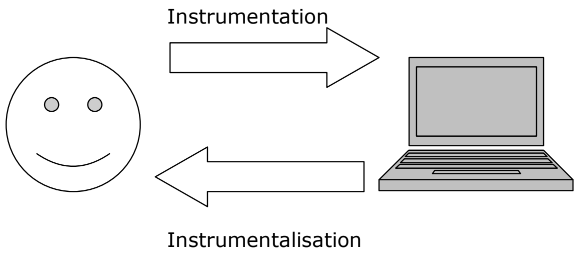

Figure 1: The instrumental approach to the use of computers for mathematics, describes the interplay between person and artefact as a bi-directional process consisting of instrumentation, directed towards the artefact, and instrumentalisation, directed towards the person.

2.2 The semiotic approach

Mathematical discourse is rich in symbols, formulas and diagrams, and Raymond Duval has developed a theoretical machinery to describe the potentials and pitfalls of working with many representations of abstract mathematical concepts (Duval, 2006).

A semiotic representation or 'sign' consists of a material signifier which stands for something: the signified. The material substance that is manipulated in order to create representations is denoted 'medium'. Examples of media are pen and paper and a computer with a specific configuration of programs.

The central aspect of the theory is transformations of semiotic representations, particularly treatments and conversions (see figure 2). Treatments are transformations inside a semiotic system, such as rephrasing a sentence or isolating x in an equation. Conversion is a transformation that changes the system, maintaining the same conceptual reference, such as going from an algebraic to a geometric representation of a line in the plane.

Mathematical objects are typically not there to be pointed at as anything but representations, sometimes even representations of a very technical nature, and mathematical objects always have more than one semiotic representation attached to it (Duval, 2006). These two facts lead to two fundamental issues in learning mathematics; (1) one common mistake is to confuse the mathematical object with one of its representations, and (2) transformation of semiotic representations can be difficult, but a lot of the creative potential in mathematics stem from transformations of semiotic representations (e.g. calculations) (Duval 2006).

Duval shows that students often have problems with changing between types of representation, particularly if this change of representational form not include a recipe for translating parts of representation in the starting register to parts of the representation in the target register. One example of a problematic change in representational form is from a plot of a function to an algebraic formulae, whereas the other way (from formula to graph) is conceptually simple since creating a table of (x,f(x)) values in principle constitute a recipe.

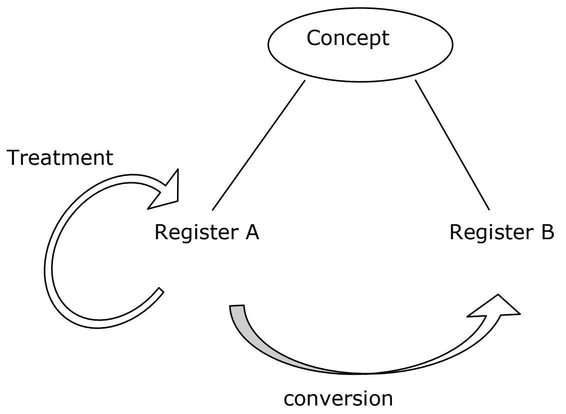

Figure 2: The transformative processes: conversion and treatment.

Conceptual understanding can be described as a person's degree of freedom in choosing various semiotic representations of the same mathematical concept (Winsløw 2003).

3. EXAMPLES OF MATHEMATICAL ACTIVITY

Representations, media and tools can affect mathematical activity. In various mathematical practices, this interplay can take different forms. Below, I describe some examples that show aspects of the relation between representations, media and tools. The first example concerns mathematicians' use of various media to support their writing process. The second is e-learning based teaching of undergraduate mathematics, and the final example is the use of dynamic geometry software in teaching of mathematics.

3.1 Researchers' mathematical writing

I have conducted an interview study among professional mathematicians (Misfeldt, 2006). The objective of the investigation was to understand the writing process of professional mathematicians from early idea to finished paper. In particular, I compared the purposes that writing served with the mathematician to the types of representations he/she used and the media (computer or pen and paper) he/she chose to use.

The result of the investigation suggests that it makes sense to consider the following five different functions of writing in mathematics (Misfeldt 2006):

Heuristic treatment consisting of getting and trying out ideas and identifying connections.

Control treatment is a deeper investigation of the heuristic ideas. It can have the form of pure control of a proposition or an open-ended investigation e.g. a calculation. It is characterised by precision.

Information storage to save information for later retrieval. Either electronically or on paper.

Communication with fellow collaborators: Ranges from annotation of an existing text over comments or ideas concerning a collaborative project to suggestions of sections to add to a paper.

Production of a paper, where writing is used to deliver a finished product intended for publication and aimed at a specific audience.

The respondents showed a strong tendency to use visual and diagrammatic types of semiotic representations to support the functions of heuristic treatment, and to some extend control treatment.

Pen and paper was dominant in the respondents' choice of medium to support heuristic and control treatment, and for some of the respondents also to support information storage. Paper is superior for heuristic treatment for two reasons. Paper is a very user-friendly technology with respect to start-up time, (screen) real estate and portability (Sellen and Harper, 2002), and paper supports a multimodal form of expression were a number of mathematical registers can be activated simultaneously (Misfeldt, 2008).

3.2 E-learning in mathematics: remediation of a flexible medium

The initiative Delta, matematik på nett, at the Norwegian technical university, consists of eight online courses in undergraduate mathematics. The topics include: Calculus, linear algebra, geometry, number theory, probability and statistics. The primary platform for the program is the learning management system 'Moodle'. The course material consists of a syllabus, brief video lectures and mandatory exercises.

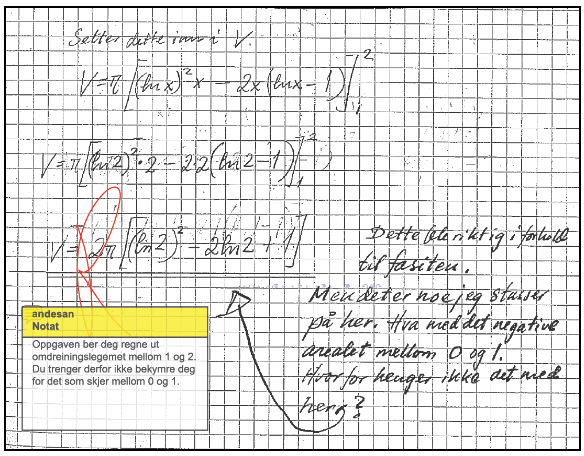

Figure 3: Teacher-student communication is mediated by handwriting.

The communication between teacher and learners about the written assignments is mediated by handwriting. The students are required to buy a scanner and scan their handwritten manuscripts in order to upload them to the learning management system, where it is reviewed and commented by the teacher, who uses an electronic pen (Misfeldt & Sanne 2007). The teachers find that this way of communication best meets their needs. The learning management system provides a discussion forum for the students', here, keyboard generated text is used to discuss mathematics. The support for mathematical notation in the discussion forum is considered insufficient by the students so they either use basic keyboard notation (such as f(x)=(3+x)/(2-x)) or tries to cut and paste formulas from other applications (Misfeldt and Sanne 2007).

3.3 Dynamical geometry: a tool that provides dynamic representations

Dynamic geometry software such as GeoGebra provides a different, more interactive and theory-related type of mathematical diagram than paper allows (Larborde, 2005). Furthermore, GeoGebra allows for a close connection between the symbolic manipulation and visualisation capabilities of computer algebra system (CAS) and the dynamic abilities of dynamic geometry software (DGS). The user can work with points, vectors, segments, lines and conic sections, but equations and coordinates can also be entered directly, and functions can be defined algebraically and then changed dynamically (Hohenwarter & Jones, 2007).

One example of GeoGebra as a "vehicle for learning" is that the concept of derivative, can be taught without the use of limits (Andresen & Misfeldt 2010).

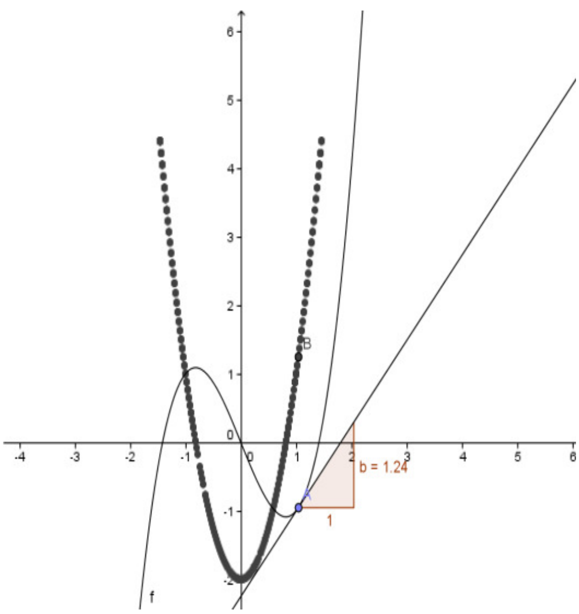

Figure 4: Teaching derivatives without limits.

In the above example, we see one way that ICT can allow a special kind of interactive representation when the teacher introduces the notion of 'derivative'. The above drawing was constructed by placing a point A on the graph of the function, and then apply the geometrical tangent to the graph of f. GeoGebra allows you to move the point around on the graph. You can then place a geometrical tangent to the graph through the point A. By defining a point B with the x-coordinate from point A and y-coordinate that is the slope of the tangent through A you can get the shape of the graph of the derivative function without considering limits at all. The example shows that the dynamic representations that the tool GeoGebra provides changes the dynamics between the geometric and algebraic register when we work with derivatives.

4. DISCUSSION

This discussion revolves around three themes: (1) Instrumental aspects of semiotic representations, (2) instrumental aspects of media, and (3) semiotic aspects of mathematical tools.

4.1 Instrumental aspects of semiotic representations

We can view semiotic representations as a specific class of artefacts that can be turned into instruments by a process of instrumental genesis. Duval (2006) shows that transformation of semiotic representations is one example of an instrumented technique applied to solve a mathematical problem. In this case, the instrument is the semiotic representations, and the development of such instrumented techniques can potentially lead to learning problems. Furthermore if the formation of viable mathematical concepts are connected to the ability to coordinate various representations of the same concept without favouring one of them, it is a relevant hypothesis that expanding of the number of representations of a concept may improve conceptual understanding.

In the example with the mathematicians' writing process, it is clear that some of the more visual and diagrammatic uses of representations belongs in the creative phase of mathematical work rather than in the communicating phase. This is an indication that these visual representations are used as mathematical instruments in a problem-solving process.

4.2 Instrumental aspects of media

In the mathematical e-learning project described, we saw how the dependence on various forms of semiotic representations affected the choice of media. They used handwritten drafts (scanned to pdf files), even though this is an unusual way to communicate electronically. This remediation was a pragmatic attempt to make the computer support handwriting, and in that sense it is a process of instrumentation, where the computer medium is controlled by the mathematical user and adapted in order to fit his/her needs. The cognitive instrument developed through the instrumental genesis is in many ways similar to handwriting. This result is interesting from an instrumental point of view, because one plausible reason for developing an instrument of 'remediated handwriting' is that handwriting has certain properties that makes it useful as a mathematical instrument.

That handwriting has some properties that allows it to serve as a mathematical instrument is also verified in the interviews with mathematicians. The mathematicians demonstrate a transition from handwriting in the early/creative part of the work towards the use of computers for information storage and communication. Again this implies that there are some aspects of pen and paper that allows it to develop into a mathematical instrument.

4.3 Semiotic aspects of mathematical instruments

To study semiotic aspects of instruments means to study how tools affect knowledge representation and designation of meaning.

The close connection between algebraic and geometric representation in GeoGebra can be described with Duval's framework. GeoGebra provides a constant double representation of mathematical concepts. This means that conversions could very well be rather different, cognitively, when you do geometry in GeoGebra, compared to paper and pencil. According to Duval, one of the most difficult aspects of learning mathematics is to learn to handle conversion between registers. However the constant presence of the two most important registers makes a potential large difference in accessibility to the mathematical topic of analytic geometry, in the "representational competence" that working with the topic requires and hence (maybe only hypothetically) in the relevance of the topic as an engine to promote general literacy. The key factor for why Geogebra is a unique piece of software lies in its semiotic abilities, for instance in the simultaneous and dynamic representation of multiple registers.

5. CONCLUSION

In this paper, I have described how computational tools and representations support mathematical activities and learning. The basic claim is that it makes sense to study (1) the semiotic aspects of mathematical tools (e.g. computational tools and physical manipulatives), (2) the instrumental aspects of semiotic representations, i.e. what kinds of instrumented techniques a specific register can allow a user to develop, and (3) the instrumental aspects of the used media, for example by looking at what types of registers, and conversions between registers, a specific media facilitates. Furthermore it makes sense to study mathematical activities that rely on uses of media and representations, as a process of developing instrumented techniques, through instrumental genesis where the mathematical worker/learner on one side changes her approach to a mathematical challenge because of she is influenced by the possibilities and constraints that media and representations provides and on the other hand takes control over the representation and media that she works with and changes it to fit personal needs.

The potential of combining a semiotic and an instrumental approach is that it can allow for at description of the way toolforthoughts (Shaffer & Clinton 2006) are used as an integrated part of mathematical activity. Representations, media and tools as disjoint aspects of mathematical activities do not tell the full story. This paper represent an attempt to combine these aspects.

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