The Xmath Partial Differentiation Algorithm

Odd Bringslid

Abstract—The Xmath eBook is being developed and algorithms into a wide range of undergraduate mathematical issues embeded in Mathematica packages are available on the web using the system webMathematica. The main purpose is to visualize mathematics in the same way as would a professor do it on the blackboard stating all intermediate steps for user defined input and then presenting solutions being easily recognized by the undegraduate student which may not always be the case using the Mathematica system directly. In this way the student may work more on a personal basis, viewing one step at a time in the solving process and then being less dependent of the professors. In this paper The Xmath Algorithm for Partial Differentiation step-by-step are presented (PD Steplet)

Index Terms— Partial Differentiation (PD), Steplet, Mathematica packages, Online calculations, Pedagogical value

1. Introduction

The use of *Mathematica* [1] in education is one of the most important areas of application. The problem however is that in education we are focusing on how problems are solved perhaps more than on the final result. Since *Mathematica* only gives the final result it will be necessary to build an application on top of *Mathematica* giving intermediate results using the methods of solving given by mathematical textbooks. It is necessary to analyze the equations in depth, Xmath then using the *Mathematica* object TreeForm to be able to extract the information needed at each level of the solution process. The algorithm is different from the algorithm used by the developers of the *Mathematica* System (D).

The Xmath algorithm will solve problems typical in mathematical teaching. General partial differentiation is implemented using standard methods, tracking the solving process in detail to be easily recognized by the students.

Manuscript received 2010. Author, assoc. prof. Odd Bringslid is with the Institute of technology, Buskerud University College, Norway (odd.bringslid@hibu.no)

2. Pedagogical value

The pedagogical value of the Xmath algorithms lies in the fact that a student may simulate solving by changing parameters and type of function. The important thing is that Xmath solves the equations as would a professor do it on the blackboard then easily being recognized by the students which is not the case using the *Mathematica* system directly [2].

3. Example

Level 1

Find the partial derivative $\frac{\partial}{\partial y}(\frac{\sin(x\cdot y)}{x^2+y^2})$

Quotient Rule

$$\frac{\partial \frac{u}{v}}{\partial y}=\frac{v\frac{\partial u}{\partial y}-u\frac{\partial v}{\partial y}}{v^2}$$

$Here\ u=\sin(x\cdot y)\ and\ v=x^2+y^2,\ Finding\ derivatives\ of\ u\ and\ v$

Level 2

Find the partial derivative $\frac{\partial \sin(x\cdot y)}{\partial y}$

Chain Rule, Composite function

$Substitute\ u=g(y)$

$$\frac{\partial f(g(y))}{\partial y}=\frac{\partial f}{\partial u}\cdot\frac{\partial u}{\partial y}$$

$Inside\ function\ u=x\cdot y\ and\ outside\ f(u)=\sin(u)$

Level 3

Find the partial derivative $\frac{\partial (x\cdot y)}{\partial y}$

Linear Rule, Constant factor

$$\frac{\partial(c\cdot f)}{\partial y}=c\cdot\frac{\partial f}{\partial y},\ Here\ c=x\ and\ \ f(y)=y$$

Finding the derivative of the non − constant factor f(y)

Level 4

Find the partial derivative $\frac{\partial y}{\partial y}$

Power Rule

$$\frac{\partial y^n}{\partial y}=n\cdot y^{n-1},\ Here\ n=1$$

$$\frac{\partial y}{\partial y}=1$$

Result, Linear Rule Constant factor

$$\frac{\partial(x\cdot y)}{\partial y}=x\frac{\partial y}{\partial y}=x$$

$$\frac{\partial u}{\partial y}=x$$

Finding the dervative of the outside function

Level 3

Find the partial derivative $\frac{\partial\sin(u)}{\partial u}$

Sin Rule

$$\frac{\partial\sin(u)}{\partial u}=\cos(u)$$

$$\frac{\partial f}{\partial u}=\cos(u)$$

Result, Chain Rule

Substitute $u=g(y)=x\cdot y$

$$\frac{\partial\sin(x\cdot y)}{\partial y}=\frac{\partial(g(y))}{\partial y}=\frac{\partial f}{\partial u}\frac{\partial u}{\partial y}=x\cdot\cos(x\cdot y)$$

This gives $\frac{\partial u}{\partial y}=x\cdot\cos(x\cdot y)$ *and* $v\frac{\partial u}{\partial y}=x(x^2+y^2)\cos(x\cdot y)$

Level 2

Find the partial derivative $\frac{\partial(x^2+y^2)}{\partial y}$

Linear Rule, Sum

$$\frac{\partial(x^2+y^2)}{\partial y}=\frac{\partial x^2}{\partial y}+\frac{\partial y^2}{\partial y}$$

Level 3

Find the partial derivative $\frac{\partial x^2}{\partial y}$

Constant Rule

Derivative of a constant is 0 (independent of y)

$$\frac{\partial x^2}{\partial y}=0$$

Result, Linear Rule Sum

$$\frac{\partial(x^2+y^2)}{\partial y}=\frac{\partial y^2}{\partial y}+\frac{\partial x^2}{\partial y}=2y$$

This gives $\frac{\partial v}{\partial y}=2y$ *and* $u\frac{\partial v}{\partial y}=2y\sin(x\cdot y)$

in the second part of numerator of the rule.

We then find the numerator:

$$v\frac{\partial u}{\partial y}-u\frac{\partial v}{\partial y}=x(x^2+y^2)\cos(x\cdot y)-2y\cdot\sin(x\cdot y)$$

Result, Quotient Rule (Answer)

$$\frac{\partial\frac{\sin(x\cdot y)}{x^2+y^2}}{\partial y}=\frac{v\frac{\partial u}{\partial y}-u\frac{\partial v}{\partial y}}{v^2}=$$

$$\frac{x(x^2+y^2)\cos(x\cdot y)-2y\sin(x\cdot y)}{(x^2+y^2)^2}$$

4. THE LINEAR RULE

The linear rule will differentiate a function like

(1) $f(x_1,x_2,..)=a_1f_1(x_1,x_2,..)+a_2f_2(x_1,x_2,..)+...$

Here a_i is independent of x. The rule is divided into the linear rule sum and the linear rule consant factor.

The function is broken down for analyzing by using the *Mathematica* object TreeForm [3] with 2 levels. The *Mathematica* object D [4] gives the derivative. Linear rule sum:

```
LinearList=Reverse[Level[TreeForm[f],2]];
LinearList=Delete[LinearList,1];
Do[Main[ak*yk,level],{j,1,Length[LinearList]}]
] (*end Do*));
Result=Sum[Main[ak*yk]]
```

Figure 1 PseudoCode Linear Rule Sum

```
FactList=Reverse[
  Level[TreeForm[a*y[x]],2]];
If [ FreeQ[FactList[[3]],x],
Result=FactList[[3]] * Main[FactList[[2]],x,level]]
```

Figure 2 PseudoCode Linear Rule Constant Factor

5. The Quotient Rule

The quotient rule will differentiate a function like

$$(2) \quad f(x_1, x_2, \ldots) = \frac{u(x_1, x_2, \ldots)}{v(x_1, x_2, \ldots)}$$

```
expr=u/v;
  u=Numerator[expr];
  v=Denominator[expr];
Result=Main[D[u,x]*v-D[v.x]*u)/v^2,level];
```

Figure 3 PseudoCode QutientRule

6. Chain Rule

This rule is used for a function being a composite function of the form y=f (g(x)). The derivative is given by the chain rule

$$u = g(x)$$

$$\frac{\partial f(g(x))}{\partial x} = \frac{\partial f(u)}{\partial u} \cdot \frac{\partial u}{\partial x}$$

```
expr=g[u[x]];
u=Reverse[Level[TreeForm[g[u[x]], 2]];
Result=Main[D[expD[g[u],u]*D[u[x]];
```

Figure 4 PseudoCode ChainRule

Main Program

PseudoCode given for rules used in the eaxmple.

```
Main[expr,x,level]:=Module[{},
   k=Reverse[Level[TreeForm[e …)xpr,2];
   Which[
      FreeQ[expr,x],DConstantRule[expr,x,level],
            expr===x,DxRule[expr,x,level],
            Head[expr]===Tan,

If[Last[k]===x,DTanRule[expr,x,level],DChainRule[expr,x,level
]],
            (* Same system for
Cos,Sin,Log,Arctan,ArcSin,ArcCos*)
      Head[expr]===Plus,DPlusRule[expr,x,level]
            Head[expr]===Times&&(Not[FreeQ[Denominator[
k[[2]]],x]]
                        v Not[FreeQ[Denominator[k[[3]]],x]]),
DQuotRule[expr,x,level],
             Head[expr]===Times&&FreeQ[Last[k],x] v
             FreeQ[k[[2]],x],DProdCoRule[expr,x,level],
      Head[expr[===Times, DProductRule[expr,x,level],


Head[expr]===Power&&k[[3]]===x&&FreeQ[k[[2]],x],DPower
Rule[expr,x,level],


Head[expr]===Power&&FreeQ[k[[2]],x],DChainRule[expr,x,lev
el],


Head[expr]===Power&&Head[k[[2]]]===Symbol&&FreeQ[k[[3]
],x],
            DDPowerRuleExp[expr,x,level],


FreeQ[k[[3]],x]&&Head[expr]=Power,DChainRuleExp[expr,x,le
vel],
    Head[expr]==Power&&Not[FreeQ[k[[2]],x]],
     LogarithmicRule[expr,x,level]
 ]
```

Figure 5 PseudoCode Main Program

The Main Program invokes the Mathemaica objects FreeQ [5], Head[6], Which[7] and Reverse [8]

Acknowledgment

Acknowledgements are expressed to the partners of the EU projects Xmath [9] and dMath [10].

References

[1] Wolfram Research, http://www.wolfram.com/
[2] Bringslid, Odd, Norstein, Anne (2008), Teaching Mathematics using Steplets. International Journal of Mathematical Education in Science and Technology 39:7 pp 925-936
[3] Wolfram Research, The Mathematica Book (2003) pp 236-237
[4] Wolfram Research, The Mathematica Book (2003) pp 80
[5] Wolfram Research, The Mathematica Book (2003) pp 124
[6] Wolfram Research, The Mathematica Book (2003) pp 231
[7] Wolfram Research, The Mathematica Book (2003) pp 345
[8] Wolfram Research, The Mathematica Book (2003) pp 127
[9] Xmath project (http://dmath.hibu.no/xmath/)
[10] dMath project (http://dmath.hibu.no)

Odd Bringslid
Assoc. Professor at The Institute of Technology, Buskerud University College, Norway. Project coordinator of EU Leonardo project dMath [1], EU Minerva project Xmath [4] and a former third party program designer for HP calculators.

Officially appointed as Project leader of Educational Government Projects. Partner in the JEM project. Conference leader of several European Conferences into eLearning. Guest Editor of the Journal of Computers in Mathematics and Science Teaching. Research into mathematical eLearning and Computer Algebra and proposed Member of the OpenMath Society.