Topological Consideration to the Development of Drawing in Children

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Abstract—*The aim of this research is to understand the structure of children's "logic in thinking" by way of analyzing their coceptions of space. It is quite unique, and different from that of Geometry. We have examined the drawings of a model by children between 3 and 5 years old. In this research we take notice of the <pairs in opposition> by children. It is shown in the drawings of children, such as open and closed, intersecting and non-intersecting, inside and outside of the contour lines and domains. We may get some hints for mathematical education from this study.*

Index Terms—*geometry, logic, cognition, domains, contour lines*

1. Introduction

In the class of mathematics many teachers are liable to give the correct answers to the children without asking them the reason. Their interest is directed not to the thinking of children but to the teaching materials in mathematical education. The result is that children can not understand sufficiently the new materials they meet and are confused by them. Teachers should have strong interest in how children think about the materials and where they get into troubles. Children are easily confronted with many troubles and failures in the trials to grasp the new materials. And a hint to understand the way of thinking by children lies in these troubles and failures. The aim of our study is to ask about the structure of their thinking by means of finding the reason of these troubles and failures.

In order to understand the development of cognition of space, we should take notice of the <pairs in opposition > in children. For example, in the case of playing pee-ka-boo, a baby knows of his/her mother not at the moment he/she recognizes her existence because he/she misses her. So by way of recognizing < the negative of her>, the baby knows < the positive of her>. There are various opposite poles about the ways of thinking in children.

But this problem has not been fully analyzed in previous researches. We will try in section eight short discussion.

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2. Task and Procedure

Why do differences occur when children draw one and the same model? We set forth a hypothesis to this question that there appear characteristics in their troubles in accordance with the stages of their development, and these characteristics are related to the topological understanding of the model.

The model (Figure1) consists of 4 domains; one blue domain in the outer side including 3 inner domains. How do children grasp the model? We have examined the drawings of Figures by 547 children (1628 sheets) from 3 to 5 years old. (Table 1: 2002.5 / 2003.5, Sendai, Japan)

a) Pretest: We asked children to draw the model without any suggestion.
b) After drawing, they played one of the following games .
GAME1: Children who were out of the closed line, throw a ball to children on the inner side of it. If a child in the circle cannot catch the ball, he must go out of the circle. He must change roles.
GAME2: When a child from one side of a whorled line met a child from the other side of it, they play the game of "paper, stone and scissors."

Figure 1: model

Table 1:number of children and sheets

	pretet		posttest	
2003	N	sheets	sheets	total
3 years old children	51	85	82	167
4 years old children	160	205	193	398
5 years old children	145	238	232	232
2002				
5 years old children	50	179		
4 years old children	141	414		
(N is number of children)				1628

The game1 and 2 are both "learning". During the game, children must distinguish the opposite

moments by themselves, such as open and closed, inside and outside of the contour lines and domains.

c) After playing the game, they were asked to draw the same model again.

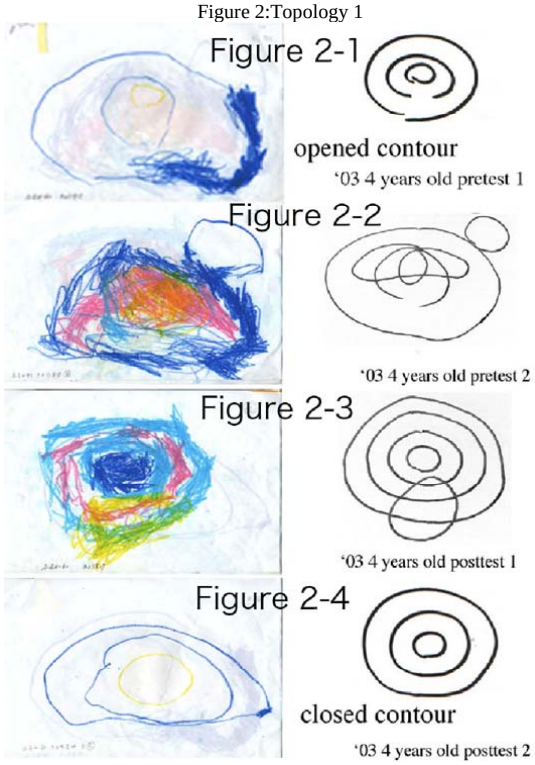
We could find changes in the drawings after they played games. We supposed the changes were caused by their own thinking and "discovery" while playing the games.

3. *OPEN AND CLOSED*

We'll take Figure 2, drawn by one and the same child. This child imitated the model by drawing 3 open contour lines. (Figure 2-1,pre test, 2003, 4 years old) The model consisted of 4 closed contour lines. Although the model has closed contour lines, he drew it with open ones.

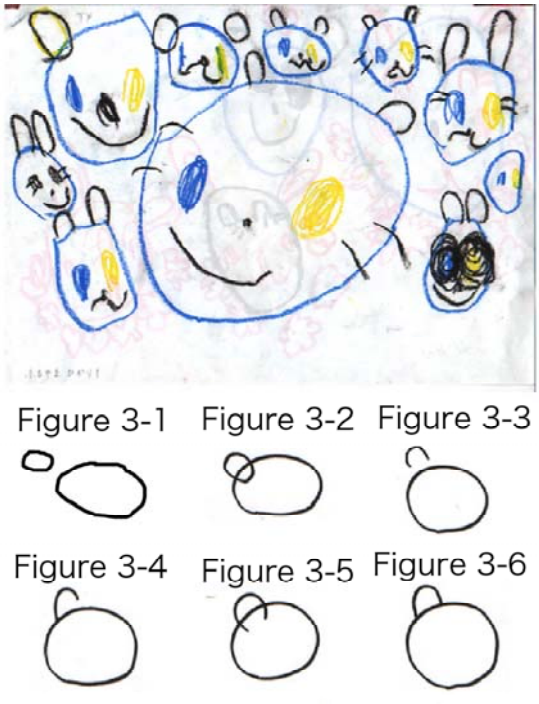
Next time he drew it, the lines were closed. (Figure 2-2, 2-3, 2-4) In order to draw the model closed, he had to think of the meaning of both the open lines and closed ones. By way of thinking of the opposite poles, he could draw them closed.

Figure 2:Topology 1



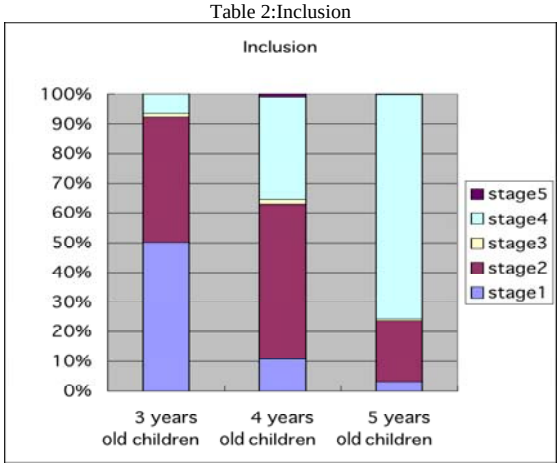
Comparing Figure 3-1 and 3-2, we can find the differences of understanding of the model by a child. This child drew the model as <a face of a bear> (Figure 3). We can take notice of the relation of the closed contour line of the face and that of the ear. The contour line of the face is drawn closed. The contour lines of the ear are drawn at the same time open and closed. Thus this child drew the lines both open and closed. (Figure 3-1~3-6) In this case he tried to vary the drowing of the ear. He might have thought about both extremes, and he'd soon find the right answer. Only by telling the correct answer to children, a teacher could not have expected to teach them the true relation.

Figure 3 :Topology 2



4. *INCLUSION*

Table 2:Inclusion



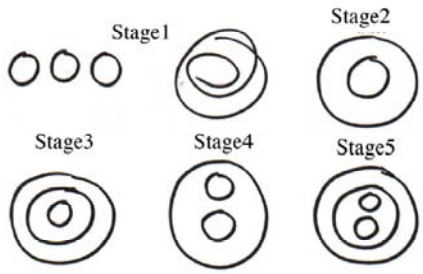
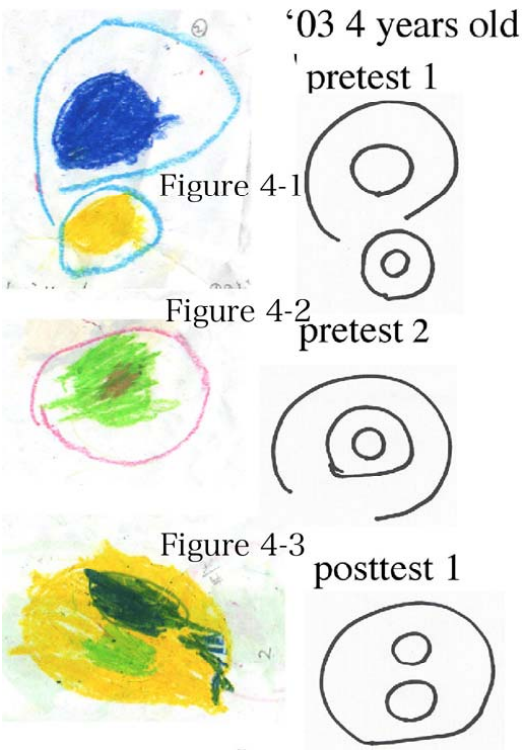


Table 2 shows the relationship of inclusion. It shows topological connections between domains. Stage 1 means that the idea of inclusion is not yet realized. In the drawing of 4 years old children,

many drawings are in Stage 2. The number of drawings in Stage 4 increases according to age. To understand Stage 4, a child must have both the idea of inclusion and non-inclusion, and it is difficult to have both ideas for children in at an earlier age.

Figure 4 shows what a child drew in the pre test as Figure 4-1 (this means Stage 2). In the post test after the play, he could draw it in Stage 3 (Figure 4- 2) and Stage 4 (Figure 4-3).

Figure 4:Topology 3



5. CONTOUR LINE

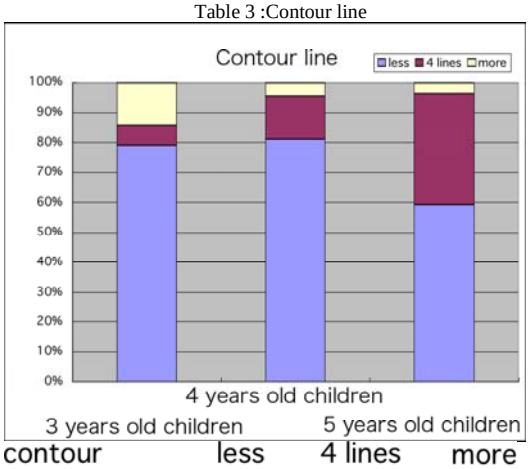
The graph in Table 3 shows how many contour lines were drawn by children. The model has 4 contour lines. For 4 years old children, it is difficult to draw the correct 4 contour lines. The rate of drawing with 4 contour lines increases by age.

In 2002, a child (4 years old) drew a drawing, such as a white domain that is enclosed by a blue contour line left empty (Figure 5-A, pre test). In the next year, she drew the correct drawing.

In the drawing 5-B the pink domain was drawn outside the blue domain. In the case of the drawing 2 (Figure 5-B, pre test), the pink domain is outeside the blue contour line. Why were the blue contour line and the pink domain left reversed?

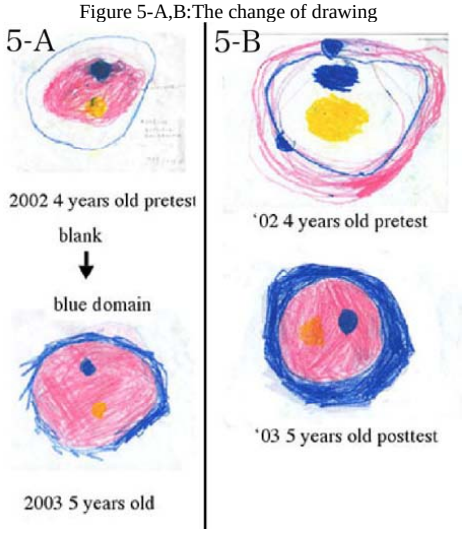
For many children the relation of contour lines and colored domains is not fully understood until they notice the opposition between contour lines and domains.

Table 3 :Contour line



contour	less	4 lines	more
3 years old children	61	5	11
4 years old children	356	64	19
5 years old children	170	107	10

Figure 5-A,B:The change of drawing

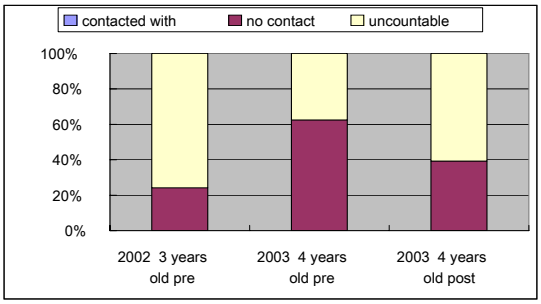


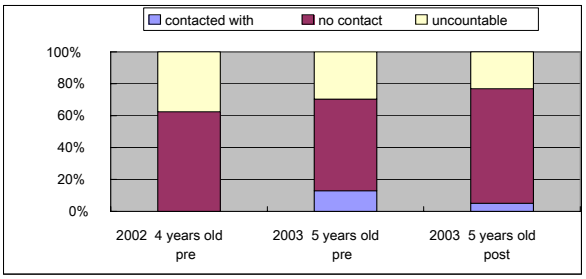
6. THE RELATION : CONTOUR LINES AND DOMAINS

Table 4 and Table 5 show the percentage of the stages of understanding between two domains. In the graph, <uncountable> means that a domain which is left undrawn exists and this domain cannot be counted (In the case of Figure 5-B, the small blue and yellow domain are not surrounded by the pink domain). <No contact> means a domain has only contour line, or even if a domain is painted, the contour line of the domain do not come into contact with the printed area.

It is difficult for children to show the relation of contour lines and domains. Most of the 3 years old children, it is difficult to draw it well, ie.the pink domain and the blue or yellow domain are not in contact with the contour lines. In 3 years old children, they grasp the pink domain and blue domains differently. They usually pay attention to the pink domain, and neglect the outer blue domain. Why? They will be fond of thinking about the complex problems (ie. pink domain which has the complex relation to other domains).

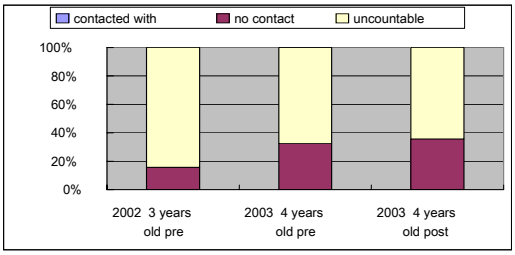
Table 4 :Contact with (pink & outer blue)



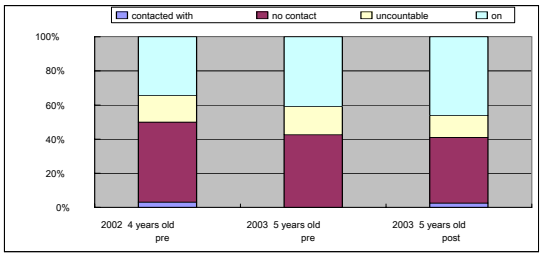


	contacted with	no contact	uncountable
2002 4 years old pre	0	40	24
2003 5 years old pre	7	31	16
2003 5 years old post	2	28	9

Table 5 :Contact with (pink & small blue)



	contacted with	no contact	uncountable
2002 3 years old pre	0	23	72
2003 4 years old pre	0	20	12
2003 4 years old post	0	11	17



	contacted with	no contact	uncountable	on
2002 4 years old pre	2	30	10	22
2003 5 years old pre	0	23	9	22
2003 5 years old post	1	15	5	18

And most children have not enough ideas about the existence of the blue domain. In the case of 4 years old children, although they drew both the pink domain and blue domains, the contour line did not contact the domain. In 5 years old children, about half of them distinguished both the pink domain and blue domains correctly.

From this result we can say that the relation of the domains and contour lines causes various kind of confusion in the thinking of children, but only if they notice “the opposite character” of them, they can draw them correctly, and their cognition of space will become consistent.

6. *THE CHANGE OF DRAWING*

A 3 years old child in 2002, drew the model as in Figure 6. Next year (2003) he drew the model as the drawings below. In Figure 6, drawings showed no change. In 2002, he drew the model relatively well as his other classmates. So he might have flattered himself that he was clever. This shows unless a 3 years old child notice his own troubles and make efforts, he will not change his thinking.

Figure 6: No change in the drawings

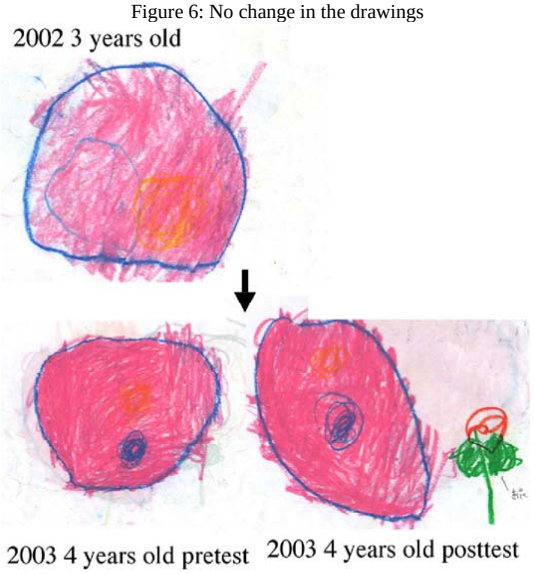
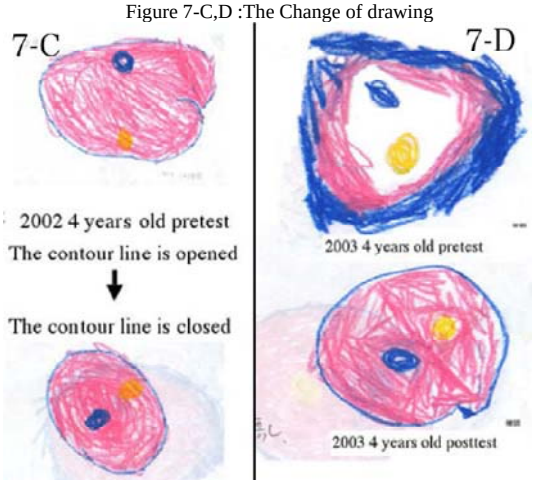


Figure 5-B shows a large change in the drawing. When the child was 4 years old, she couldn’t notice the relation of domains and drew conversely. Next year she could draw the relation between the domains.

In Figure 7-C, he showed a small but not unimportant change in the drawing, he drew the contour line from open to closed.

In Figure 7-D, he drew the model in which the blue domain has no contour line and small blue and yellow domain do not have contact with the pink domain. In the post test, though there is no blue domain,the pink domain contacts with the small blue and yellow domain. In this case he made some progress and lost some understanding.

Figure 7-C,D :The Change of drawing



8 WHAT IS NEW COMPARED TO PRIOR RESEARCHES ?

Our attempt has been carried out under three perspectives. First, by analyzing the drawings of geometrical figures by children with a topological method, we have intented to make the degrees of development in the cognition of space efficiently clear. Secondly, we took notice of the fact that the troubles and failures by children have 'logical cause', namely the lack of "an pairs in opposition ". Troubles and failures are not caused by accident or individualy. If they find the "pairs in opposition " by themselves, they will make great progress in learning. Third, one of the largest problems in the cognition of space by children is to distinguish the relations between contour lines and domains. If they clear this problem, they will acquire a stabilized cognition of space.

We are convinced that these three points have not yet been fully discussed. These points have been easily understood in the investigation into the cognition of space by children. In the classical study, "*The Child's Conception of Space*"(1948), J. Piaget and B. Inhelder [1] thought that the concept of space by children develops from a simple topological type to projective and euclidean. "We shall find that the child's space, ⋯invariably begins with this simple topological type of relationship long before it becomes projective or euclidean."(p. vii) But this understanding is not the case. Children don't go from easy to complex matters. They rather show their interest in complex problems and get into trouble without having any 'logic for solution'. Children always lunge out toward unknown world. When they find their 'logic for solution', they will make great progress, and this is also the case in their cognition of space.

Z. P. Dienes and E. W. Golding [2] had introduced some practical methods in the education of the cognition of space. (*Exploration of Space and Practical Measurement*, 1966) They thought that "a child, from the moment he is born, explores space", and they tried to devise many geometrical games using a topological method. But, I think, they didn't appreciate enough the role of trouble and failure in children, and applied directly a mathematical procedure to the result of games.

So we have tried to elucidate 'the structure of troubles', and introduced the method of combination of drawings and games, and to establish some criteria for the appraisal of development.

Recently a lot of researches is being carried out in the field of drawing-analysis for the understanding of the stages of develpomentment in children. It is natural that these attempts ware usualy made from the interest of cognitive psychology. For exemple, Analice Dutre Pillar [3] tried to demonstrate the relation between the stages of drawing and that of their <action and thought>. Aaro Toomela [4] tried to understand the development of children by the stages of drawing. According to her, drawings by a child develop with age from the category of 'scribble' to that of 'integrated whole'. But by this explanation we can't understand why and by what he/she changes the categories. To this problem asserted Annie Vinter [5/6] to pay attention to 'read' the drawing, and suggested to inquire '*how* lebel' of the problem. Children can be 'flexible and open to semantic influence', manage to meet the problem. Thus they grow from global sensibility to meaning. She also tried to clear the problem of 'implicit learning' using the method of drawing, and concluded that implicit learning is not related to age, but to the behavior manipulation. Similar issue was tried by I.A.Apperly ,E.Williams, and J. Williams [7]. They took notice of the role of symbols for the acquirement of stable meaning by children.

Investigations to clear the development of children by means of drawing are adopted to other theme, for exemple to the problem of emotion caused by colour [8], to that of learning from other peoples mistakes [9], to that of autism or learning disabilities [10], or to the exploitation of computational model [11]. For us it is also worth noticing that in the field of mathematical education many researches are made applicating figure drawing [12/13].

To these researches, our trial make much more of the role of topology for the illumination of drawing ability in children, especialy of the relation of domains and contour lines. One of the greatest trap for children lies certainly in this moment.

9. CONCLUSION

1 Development of learning in the cognition of space begins from noticing their own trouble and failure in understanding about the relation of domains and contour lines .

2 It is important for teachers to encourage a structured and systematic way of the thinking in the children.

We take notice of the <pairs in opposition > in the drawing and playing by children. Especially in the drawing, opposite problems such as open and closed, intersecting and non-intersecting, inside and outside play important roles for the overcoming of troubles by children.

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