Bit and Power Loading for Wireline Multicarrier Transmission Systems

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Abstract—*The partitioning of transmit power and the allocation of bits per symbol to the modulation schemes within the subchannels of multicarrier systems essentially affect their performance capability. Here a two-stage optimization setup is proposed: Firstly, the bit rate is maximized using the Lagrange Multiplier method, which in general leads to non-integer numbers of signalling levels. Based on given practical constraints by e. g. the rounding of the number of signalling levels, an interesting extension is investigated: A second optimization step is applied to improve the system performance further in terms of decreasing the bit-error rate or in terms of increasing the bit rate. For the rounding of the number of signalling levels two different approaches are investigated. Exemplary results are obtained for the multicarrier transmission over twisted wire pairs.*

Index Terms—*Multicarrier transmission, power allocation, bit loading, filtered multitone modulation, Lagrange multiplier method, quadrature amplitude modulation.*

1. Introduction

MULTICARRIER transmission techniques are powerful alternatives compared with single-carrier or baseband transmission techniques [3], [4]. On the one hand they are used for the transmission over copper cables with strong frequency-dependent attenuation in the local cable area (e. g. ADSL, asymmetric digital subscriber line) and on the other hand they are successfully applied in digital transmission systems for frequency-selective radio channels (e. g. DVB, digital video broadcasting or DAB, digital audio broadcasting). Multicarrier systems divide the available frequency range into narrow subchannels. The allocation of signalling levels (i. e. bits per symbol) and the partitioning of transmit power to the subchannels are degrees of freedom, which essentially affect the performance capability of multicarrier transmission systems [5].

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Known bit loading and transmit power allocation techniques are often based on a fixed target bit rate. Examples can be found in a huge variety in the literature: The allocation scheme according to [6] allocates the signalling levels of the subchannel modulation schemes based on the channel capacity. The transmit power after rounding the signalling levels towards integer numbers of bits per symbol is assigned in such a way that equal error rates in the subchannels arise. The approach according to [7] is based on maximizing the signal-to-noise ratios per subchannel. The algorithm [8] successively assigns the signalling levels to the subchannels in such a way that the next bit per symbol is allocated to the subchannel, which requires the least additional transmit power until the target bit rate is reached. The aim of these methods is an increase of the transmission quality at a fixed overall bit rate. This is particularly appropriate, when the reliability of a transmission at e. g. a standardized bit rate should be increased or a transmit power reduction is desired at a given bit-error rate and a fixed bit rate.

In this contribution the allocation of the number of signalling levels and the transmit power to the subchannels is considered under the aspects of maximizing the overall bit rate at a fixed required quality (e. g. bit-error rate or signal-to-noise ratio) or of minimizing the bit-error rate at a given bit rate. The former is of great practical interest in wireline and wireless transmission, respectively, since the demand for high data rates is growing very fast. The latter one is often required, if the transmission reliability has to be improved at a standardized bit rate.

The intention of this contribution is not to develop a new bit and power loading algorithm, which outperformes existing ones. Here, general aspects and interrelationships of bit and power allocation in multicarrier systems are considered. The effects of complementary optimization targets such as bit rate maximization or bit-error rate minimization are investigated under given (fixed) boundary conditions.

The contribution is organized as follows: In section 2 the transmission model is introduced and in section 3 quality criteria and possible directions of optimization are briefly reviewed. In section 4 and 5 different power and bit allocation schemes are investigated, where the focus lies on a two-stage optimization scheme in order to make efficient use of the transmit power. In section 6 numerical results are presented. Some concluding remarks are given in section 7.

2. System model

The considered N channel multicarrier transmission system is shown in Fig. 1. The transmitter consists of N bandpass filters, whereas the first subchannel is not executed as a baseband channel but also as a bandpass channel in order to allow a unique description and enable the usage of the low frequency range for the analog telephone transmission (e. g. voice) [9]. The subchannel's

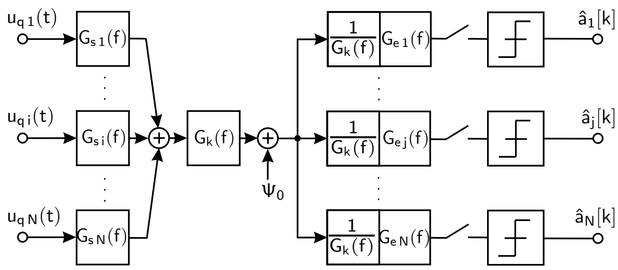

Fig. 1. Multicarrier transmission system

source signal is described by

$$u_{\mathrm{q}\,\mu}(t) = U_{\mathrm{s}\,\mu}\, T_{\mathrm{s}} \sum_{k=-\infty}^{+\infty} a_\mu[k] \cdot \delta(t - k\, T_{\mathrm{s}}) \quad , \qquad (1)$$

whereby a sequence of Dirac pulses is weighted by the amplitude coefficients $a_\mu[k]$. The value $U_{\mathrm{s}\,\mu}$ indicates the half-level transmit amplitude and the symbol duration is denoted by T_{s}, respectively [2]. Furthermore the pulse shaping concept is based on the filtered multitone modulation (FMT). Transmit and receive filters $G_{\mathrm{s}\,\mu}(f)$ and $G_{\mathrm{e}\,\mu}(f)$, respectively, are two-fold lapped base functions as proposed in [10] and investigated in [9]. Compared with a multicarrier system, which uses a rectangular pulse shaping (e. g. OFDM, orthogonal frequency division multiplex) a guard interval can be avoided here. This leads to an increase in the spectral efficiency with the drawback of a slightly increased complexity (e. g. equalization unit) [11]. The influence of different pulse shaping concepts (e. g. FMT, Wavelet) and their behaviour under non-ideal channel conditions were investigated in [12] and [9].

The cable is described by the transfer function

$$G_{\mathrm{k}}(f) \approx \frac{1}{2} \frac{1}{\prod\limits_{\nu=1}^{\infty} \left(1 + \mathrm{j}\frac{f}{f_\nu}\right)} \text{ with } f_\nu = \frac{\pi^2\,(2\,\nu-1)^2}{4\,l^2} f_0 \quad , \qquad (2)$$

where l denotes the cable length (in km) and f_0 is the characteristic cable frequency (in $\mathrm{MHz}\cdot\mathrm{km}^2$), respectively [13]. Based on the choice of the combined equalizer-receive filter function $G_{\mathrm{e}\,\mu}(f)/G_{\mathrm{k}}(f)$, intersymbol and interchannel interference can be avoided completely (Fig. 1). Other equalization concepts in combination with FMT modulation were investigated exemplarily in [14] or [12].

3. Quality criteria and optimization setups

The quality of the data transmission can be evaluated by using the well-known signal-to-noise ratio (SNR) definition (e. g. [12], [15])

$$\varrho = \frac{(\text{Half vertical eye opening})^2}{\text{Noise disturbance}} \quad . \qquad (3)$$

With this definition a SNR per subchannel can be defined in the following form:

$$\varrho_\mu = \frac{U_{\mathrm{s}\,\mu}^2}{U_{\mathrm{R}\,\mu}^2} \qquad \mu = 1, 2, \cdots, N \quad . \qquad (4)$$

Here, the half vertical eye opening at the sampling instant is identical with the half-level transmit amplitude $U_{\mathrm{s}\,\mu}$ since intersymbol and interchannel interference can be avoided completely after receive filtering (with equalization) and sampling (Fig. 1). Further, the noise disturbance per quadrature component after receive filtering and sampling is denoted by $U_{\mathrm{R}\,\mu}^2$ [12]. With this definition a symbol-error rate per subchannel using QAM (quadrature amplitude modulation, e. g. [16], [17]) can now be formulated as a function of the number of signal points M_μ and the SNR ϱ_μ

$$P_{\mathrm{f}\,\mu} \approx 2\left(1 - \frac{1}{\sqrt{M_\mu}}\right) \mathrm{erfc}\left(\sqrt{\frac{\varrho_\mu}{2}}\right) \quad . \qquad (5)$$

Assuming gray coding [17], the bit-error rate per subchannel yields to

$$P_{\mathrm{b}\,\mu} = \frac{1}{\log_2(M_\mu)} P_{\mathrm{f}\,\mu} \quad . \qquad (6)$$

Combining (5) and (6), the bit-error rate of the whole multicarrier system can be evaluated by taking all subchannel bit-error rates $P_{\mathrm{b}\,\mu}$ into account and results in

$$P_{\mathrm{b}} = \frac{1}{N} \sum_{\mu=1}^{N} \frac{2}{\log_2(M_\mu)} \left(1 - \frac{1}{\sqrt{M_\mu}}\right) \mathrm{erfc}\left(\sqrt{\frac{\varrho_\mu}{2}}\right) \quad . \qquad (7)$$

The optimization requires not only the consideration of the bit-error rate but also the data-rate and their mutual impact on each other. In many cases the data throughput

$$f_{\mathrm{B\,ges}} = \sum_{\mu=1}^{N} f_{\mathrm{B}\,\mu} = f_{\mathrm{T}} \sum_{\mu=1}^{N} \log_2(M_\mu) \qquad (8)$$

has to be maximized, where the boundary conditions of a restricted transmit power P_s and a required error-rate $P_{\mathrm{b\,ref}}$ have to be fulfilled. These constraints can be defined as follows

$$\text{a)}\quad P_\mathrm{s}-\sum_{\mu=1}^{N} P_{\mathrm{s}\,\mu} \geq 0 \quad \text{and} \quad \text{b)}\quad P_{\mathrm{b\,ref}}-P_\mathrm{b} \geq 0\ . \tag{9}$$

Throughout this contribution it is assumed that the number of subchannels N and the total transmit power P_s are fixed. The remaining parameters like the symbol pulse frequency $f_\mathrm{T} = 1/T_\mathrm{s}$, the partitioning of the transmit power $P_{\mathrm{s}\,\mu}$ to the subchannels and the expected error-rate $P_{\mathrm{b\,ref}}$ as well as ϱ_μ and M_μ are degrees of freedom. Unfortunately they are not independent and mostly have a strong impact on each other. A cost function can now be defined

$$\begin{aligned} J &= f_{\mathrm{B\,ges}} + \lambda_1 \left(\sum_{\mu=1}^{N} P_{\mathrm{s}\,\mu} - P_\mathrm{s} \right) \dots \\ &\quad \dots + \lambda_2 \left(\frac{1}{N} \sum_{\mu=1}^{N} P_{\mathrm{b}\,\mu} - P_{\mathrm{b\,ref}} \right) \end{aligned} \tag{10}$$

using the Lagrange multiplier method, where λ_1 and λ_2 are the Lagrange multipliers [18]. Unfortunately the solution of this cost function is highly complex. That is why in most cases only one of the two boundary conditions is considered in the optimization process by the search for closed-form analytic solutions. The use of the Lagrange multiplier method in combination with power allocation schemas has already been considered in a lot of publictions for wireline and wireless channels, respectively [19]–[23].

4. Throughput maximization at a fixed SNR per subchannel

A. Basics

Ignoring the boundary condition error-rate defined in (9 b), the cost function defined in (10) can be simplified to

$$J(M_1, M_2, \cdots, M_N) = f_\mathrm{T} \sum_{\mu=1}^{N} \log_2(M_\mu) + \lambda \left(\sum_{\mu=1}^{N} P_{\mathrm{s}\,\mu} - P_\mathrm{s} \right)\ , \tag{11}$$

whereas the number of signal points M_μ should be chosen in such a way that the resulting bit rate $f_{\mathrm{B\,ges}}$ is maximized and the constraint of a restricted transmit power is maintained. In order to find a closed-form analytic solution a fixed SNR per subchannel ϱ_μ and a fixed symbol pulse frequency f_T are assumed.

Considering square signal constellations (e. g. 4-QAM, 16-QAM, 64-QAM) the transmit power per subchannel $P_{\mathrm{s}\,\mu}$ according to [24] or [16] yields to

$$P_{\mathrm{s}\,\mu} = \frac{2}{3}\, U_{\mathrm{s}\,\mu}^2\, (M_\mu - 1)\ . \tag{12}$$

For cross constellations (e. g. 32-QAM, 128-QAM, 512-QAM) the transmit power yields to

$$P_{\mathrm{s}\,\mu} = \frac{1}{48}\, U_{\mathrm{s}\,\mu}^2\, (31\, M_\mu - 32)\ . \tag{13}$$

Since the error between (12) and (13) can be neglected with increasing M (e. g. $M \geq 32$) only (12) is used for further calculations. Figure 2 shows on the one hand the exakt powers of the signal constellations (calculated according to (12) for square constellations and to (13) for cross constellations) and on the other hand the approximate powers calculated according to (12) for all constellations. It becomes obvious, that the equation (12) provides a good upper bound for the power of cross constellations (13) and for square constellations it is the exact solution. Furthermore we exclude the 8-QAM from our investigations, since both equations (12) and (13) do not consider the transmit power of an 8-QAM accurate enough. In a practical system the usage of an 8-QAM should be possible if a separate calculation of the transmit power is being done [24].

Therefore with the assumption of a given SNR per subchannel ϱ_μ the transmit power calculations simplify with (4) to

$$P_{\mathrm{s}\,\mu} = \frac{2}{3}\, \varrho_\mu\, U_{\mathrm{R}\,\mu}^2\, (M_\mu - 1)\ . \tag{14}$$

The derivation of (11) with respect to the searched parameter M_μ leads to

$$\frac{\partial}{\partial M_\mu} J(M_1, M_2, \cdots, M_N) = \frac{f_T}{M_\mu \ln(2)} + \lambda\, \frac{2}{3}\, \varrho_\mu\, U_{\mathrm{R}\,\mu}^2\ . \tag{15}$$

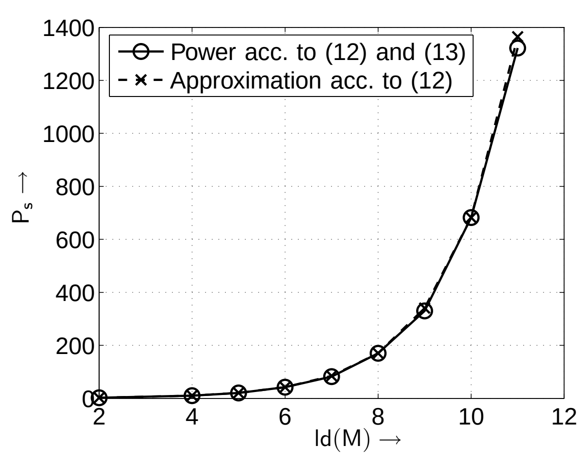

Fig. 2. QAM transmit powers as functions of the number of bits per symbol $\mathrm{ld}(M)$ (example: $U_\mathrm{s} = 1\,\mathrm{V}$)

With the assumption of an equal SNR per subchannel i. e.

$$\varrho_\mu = \varrho_0 \quad \text{for} \quad \mu = 1, 2, \cdots, N \tag{16}$$

and by setting (15) to zero, the searched M_μ are obtained:

$$M_\mu = -\frac{3}{2\,\ln(2)}\frac{f_{\rm T}}{\lambda\,\varrho_0\,U_{{\rm R}\,\mu}^2} \qquad \mu = 1, 2, \cdots, N \ . \tag{17}$$

The Lagrange multiplier λ can be determined with (9 a), (14) and (17) and results in

$$\lambda = -\frac{3\,N\,f_{\rm T}}{3\,P_{\rm s}\,\ln(2) + 2\,\ln(2)\,\varrho_0\,\sum\limits_{\nu=1}^{N} U_{{\rm R}\,\nu}^2} \ . \tag{18}$$

This solution now leads to the optimal real-valued numbers of signal points per subchannel

$$M_\mu = \frac{3\,P_{\rm s}}{2\,N\,\varrho_0\,U_{{\rm R}\,\mu}^2} + \frac{1}{N\,U_{{\rm R}\,\mu}^2}\cdot\sum_{\nu=1}^{N} U_{{\rm R}\,\nu}^2 \ . \tag{19}$$

Evaluating the expression $M_\mu\,U_{{\rm R}\,\mu}^2$, an important result for the multicarrier system design can be obtained as shown in [7]: With (19) it yields to

$$M_\mu\,U_{{\rm R}\,\mu}^2 = \frac{3\,P_{\rm s}}{2\,N\,\varrho_0} + \frac{1}{N}\cdot\sum_{\nu=1}^{N} U_{{\rm R}\,\nu}^2 = \text{constant} \ . \tag{20}$$

A capacity maximization requires a fixed $M_\mu\,U_{{\rm R}\,\mu}^2$ for all subchannels of the multicarrier transmission system with equal SNRs per subchannel as well as fixed $P_{\rm s}$, N and $U_{{\rm R}\,\mu}^2$ (based on a fixed $f_{\rm T}$). With (14) and (16) this leads to a transmit power per subchannel

$$P_{{\rm s}\,\mu} = \frac{2}{3}\,\varrho_0\,U_{{\rm R}\,\mu}^2\,M_\mu - \frac{2}{3}\,\varrho_0\,U_{{\rm R}\,\mu}^2 \ . \tag{21}$$

Here, the first term is fixed with the result formulated in (20) and also the second one for a given $f_{\rm T}$, respectively. The capacity maximization therefore requires a uniform distribution of the transmit power to the subchannels.

B. Realization

The usage of realizable QAM setups requires a roundoff of the calculated optimum M_μ. In order to achieve required quality criteria the following two rounding operations were considered:

1) Non-iterative rounding: The choice of the next smallest power of two as a practically favorable number of signal points leads to the following integer number of bits per symbol and can be described mathematically as

$$\log_2(\mathbb{M}_\mu) = \lfloor \log_2(M_\mu) \rfloor \ . \tag{22}$$

The expression $\lfloor\cdot\rfloor$ delivers the next smallest integer value.

2) Iterative rounding: Next to the considered non-iterative rounding an iterative rounding was introduced as shown in [7] in order to make a better use of the system parameters. Here, the rounding is based on the following rule:

$$\log_2(\mathbb{M}_\mu) = \begin{cases} \lfloor \log_2(M_\mu) + 0.5 \rfloor & \log_2(M_\mu) > 1.5 \\ 0 & \text{otherwise} \end{cases} \tag{23}$$

whereby the minimum number of bits per symbol was limited to a value of two (based on the assumption in section 4.A). In cases where the calculated transmit power exceeds the given total transmit power the largest rate $\log_2(\mathbb{M}_\mu)$ is, according to [7], decreased by a value of one until the calculated transmit power is lower than the given total transmit power (e. g. $P_{\rm s} = 1\,{\rm V}^2$).

C. Effects for further optimizations

These rounding operations combined with the transmit power adjustment do not have an influence on the SNR per subchannel ϱ_μ, since the SNR can be expressed as

$$\varrho_\mu = \frac{3\,P_{{\rm s}\,\mu}}{2\,(\mathbb{M}_\mu - 1)\,U_{{\rm R}\,\mu}^2} \equiv \varrho_0 \tag{24}$$

and there exists a linear relationship between $P_{{\rm s}\,\mu}$ and $\mathbb{M}_\mu$. The transmit power per subchannel can now be calculated according to

$$P_{{\rm s}\,\mu} = \frac{2}{3}\,U_{{\rm s}\,\mu}^2\,(\mathbb{M}_\mu - 1) = \frac{2}{3}\,\varrho_0\,U_{{\rm R}\,\mu}^2(\mathbb{M}_\mu - 1) \ . \tag{25}$$

Upholding an equal SNR for all subchannels $\varrho_\mu = \varrho_0$ and taking (4) into account, the reserve in the transmit power

$$\Delta\,P_{\rm s} = P_{\rm s} - \frac{2}{3}\,\varrho_0\,\sum_{\mu=1}^{N} U_{{\rm R}\,\mu}^2\,(\mathbb{M}_\mu - 1) \tag{26}$$

is obtained. This power reserve depends on the constellation sizes $\mathbb{M}_\mu$, on the noise power $U_{{\rm R}\,\mu}^2$ (via the pulse frequency $f_{\rm T}$) and on the desired SNR ϱ_0. The influence of the SNR on the arising power reserve at a fixed pulse frequency is illustrated in Fig. 3 and this power reserve can now be used in two ways in a second step of optimization: Either the throughput with an integer number of bits per symbol can be maximized or the bit-error rate at a fixed data rate can be minimized. These two ways are shown in the next section.

5. Mutual impact between bit-error rate and data-rate

A. Throughput maximization

With given constellation sizes and equal SNRs per subchannel, the resulting transmit power reserve (26) can be used to increase the overall

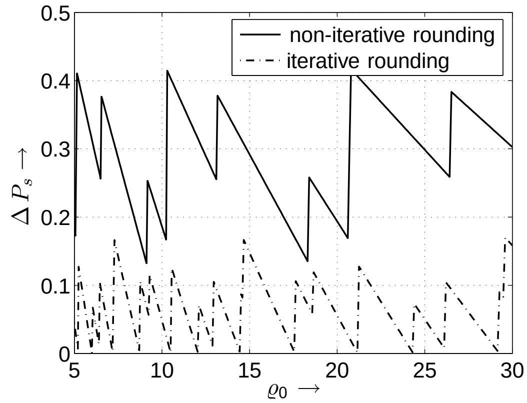

Fig. 3. Remaining power reserve ΔP_s as a function of the SNR at a fixed pulse frequency ($f_\mathrm{T} = 500$ kHz) using different rounding operations

bit rate via an increased pulse frequency. With a given integer number of bits per symbol the bit rate yields to

$$f_{\mathrm{B\,ges}} = f_\mathrm{T} \sum_{\mu=1}^{N} \log_2(\mathbb{M}_\mu) \ . \tag{27}$$

Assuming fixed values of P_s, ϱ_0 and $\mathbb{M}_\mu$, the pulse frequency f_T can now be increased and therefore the noise power $U^2_{\mathrm{R}\,\mu}$ is enhanced, until the boundary condition of a given total transmit power (9 a) is met. The expression $\varrho_0 = U^2_{\mathrm{s}\,\mu}/U^2_{\mathrm{R}\,\mu}$ remains constant in the case of a rising pulse frequency f_T, because the expression (24) is valid and there exists a linear relationship (25) between $P_{\mathrm{s}\,\mu}$ and $U^2_{\mathrm{R}\,\mu}$ via the factor $2/3\,\varrho_0\,(\mathbb{M}_\mu - 1)$.

Only the constellation size will affect the bit-error rate differently. Upholding an equal SNR for all subchannels, the differences in the bit-error characteristic will be determined by the parameter

$$\frac{2}{\log_2(\mathbb{M}_\mu)} \left(1 - \frac{1}{\sqrt{\mathbb{M}_\mu}}\right) \ , \tag{28}$$

which depends on the QAM subchannel constellation size $\mathbb{M}_\mu$. Therefore an equal SNR power allocation scheme cannot lead to the best possible bit-error rate, since the largest subchannel bit-error rate will dominate the overall bit-error characteristic.

B. Bit-error rate minimization

The transmit power reserve (26) originating in rounding the number of signalling levels can also be used to increase the overall bit rate, as shown in the preceding section. Alternatively, this power reserve can be used to correct the SNR per subchannel in order to minimize the overall bit-error rate (at a fixed f_T), now allowing different SNRs per subchannel. Starting from the bit-error rate per subchannel according to (5) and (6)

$$P_{\mathrm{b}\,\mu} = \frac{2}{\log_2(\mathbb{M}_\mu)} \left(1 - \frac{1}{\sqrt{\mathbb{M}_\mu}}\right) \mathrm{erfc}\left(\sqrt{\frac{\varrho_\mu}{2}}\right) \ , \tag{29}$$

the aggregate bit-error rate (7) has to be minimized with respect to a given total transmit power (9 a). The Lagrangian multiplier method leads to the cost function

$$J(U_{\mathrm{s}\,1}, U_{\mathrm{s}\,2}, \cdots, U_{\mathrm{s}\,N}) = \frac{1}{N} \sum_{\mu=1}^{N} P_{\mathrm{b}\,\mu} + \lambda \left(\sum_{\mu=1}^{N} P_{\mathrm{s}\,\mu} - P_\mathrm{s}\right) \ , \tag{30}$$

whereby the half-level amplitudes $U_{\mathrm{s}\,\mu}$ should now be chosen in such a way that the overall bit-error rate is minimized. Contrary to the number of bits per symbol calculated in the first step of the optimization, here the half-level amplitudes are searched. The transmit power per subchannel can be calculated via (25) for given QAM constellation sizes $\mathbb{M}_\mu$.

The derivation of the cost function with respect to the searched $U_{\mathrm{s}\,\mu}$ and setting it zero leads to the half-level amplitude of the μth subchannel

$$U_{\mathrm{s}\,\mu} = \frac{3\ln(2) \cdot \mathrm{e}^{-\frac{1}{2}A}}{\sqrt{2\pi} N \lambda \ln(M_\mu) \cdot U_{\mathrm{R}\,\mu} \cdot (M_\mu + \sqrt{M_\mu})} \tag{31}$$

with

$$A = \mathrm{W}\left(\frac{9\ln^2(2)}{2\pi N^2 \lambda^2 \ln^2(M_\mu) \cdot U^4_{\mathrm{R}\,\mu} \cdot (M_\mu + \sqrt{M_\mu})^2}\right) \ , \tag{32}$$

where $\mathrm{W}(x)$ denotes the Lambert W function [25]. Inserting these values into the boundary condition (9 a)

$$P_\mathrm{s} - \frac{2}{3} \sum_{\mu=1}^{N} U^2_{\mathrm{s}\,\mu} \left(\mathbb{M}_\mu - 1\right) = 0 \tag{33}$$

results in λ and the searched half-level amplitudes. This optimal set of half-level amplitudes for a fixed f_T (and therefore fixed $U^2_{\mathrm{R}\,\mu}$) and $\mathbb{M}_\mu$ leads to a minimal overall bit-error rate after (7) at a fixed data rate $f_{\mathrm{B\,ges}}$.

6. Results

For the following numerical evaluation the exemplary parameters $\varrho_0 = 30$ (in the first optimization step), $P_\mathrm{s} = 1\,\mathrm{V}^2, N = 4$, $l = 2\,\mathrm{km}$, $f_0 = 0.178\,\mathrm{MHz}\cdot\mathrm{km}^2$ (cable with a wire diameter of $0.6\,\mathrm{mm}$) were assumed. A white Gaussian noise with a power spectral density $\Psi_0 = 10^{-12}\mathrm{V}^2/\mathrm{Hz}$ is added at the cable output [26].

Figure 4 shows the influence of both rounding operations (iterative and non-iterative) on the achievable bit rate, when the bit rate is maximized: In both cases the maximum bit rate occurs at the same optimum pulse frequency. According to (27) the overall bit rate can be maximized either by increasing the pulse frequency or by rising the sum of the numbers of bits per symbol in the subchannels. Thus, these variables are interchangeable in the sense of maximizing the overall bit rate: In case of iterative rounding the transmit power reserve (Fig. 3) is lower, but higher numbers of bits per symbol in the subchannels can be used, whereas in case of non-iterative rounding smaller numbers of bits per symbol are used, but the higher transmit power reserve allows a variation of the pulse frequency to higher frequencies. At the (fixed) optimum pulse frequency the same numbers of bits per symbol in the subchannels are obtained according to both rounding approaches for a maximum overall bit rate. The bit-error rates per subchannel are slightly different assuming a constant SNR in all subchannels, because of the dependency of the subchannel BERs on the number of signalling levels according to (28).

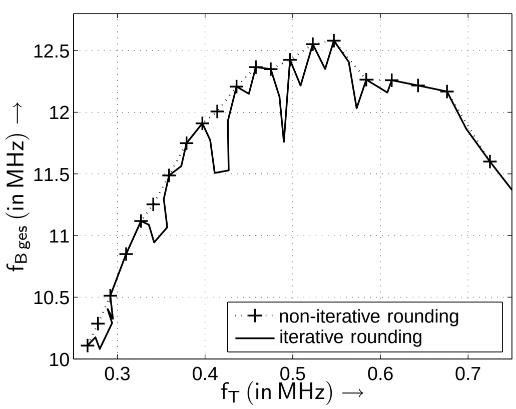

Fig. 4. Overall bit rate $f_{\mathrm{B\,ges}}$ as a function of the pulse frequency f_{T} for both rounding approaches in the first optimization step and bit-rate maximization in the second optimzation step

Figure 5 shows the overall bit rate $f_{\mathrm{B\,ges}}$ depending on the pulse frequency f_{T} for both investigated rounding operations and both strategies of optimization in the second stage (i. e. maximization of the overall bit rate and minimization of the aggregate bit-error rate for a fixed set of QAM bits/symbol in the subchannels).

Using non-iterative and iterative rounding, different amounts of transmit power reserves emerge. In case of BER minimization and non-iterative rounding, the relatively high transmit power reserve can be used for minimizing the BER in the second optimization step: That is why a smaller overall bit-error rate is obtained than in case of iterative rounding (Fig. 6), where a smaller transmit power reserve remains (at a higher overall bit

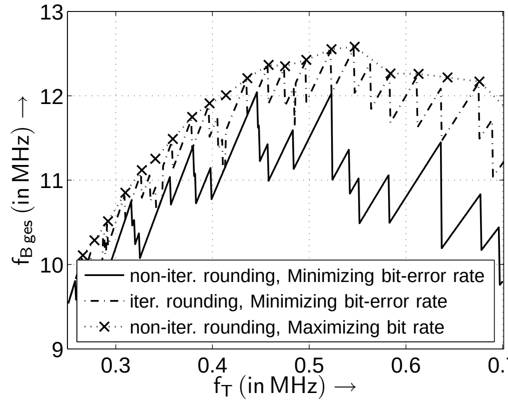

Fig. 5. Overall bit rate $f_{\mathrm{B\,ges}}$ as a function of the pulse frequency f_{T} for both power allocation approaches

rate).

7. Conclusion

In this contribution, strategies for allocating the transmit power and the numbers of bits per symbol to the subchannels of a multicarrier system using QAM were investigated. The main focus in this publication was to show the relationship between both aims of optimization: BER minimization and capacity maximization. Therefore a clearly defined reference system was considered.

Firstly, the numbers of bits per symbol were allocated to the subchannels based on an equal signal-to-noise ratio in all subchannels with the aim of a maximum overall bit rate. Here it could be shown, that a uniform distribution of the transmit power is necessary. From this, real numbers of signaling points of QAM constellations were obtained. For practical applications, the constellation sizes are rounded towards integer numbers of bits per symbol. Here, different rounding operations

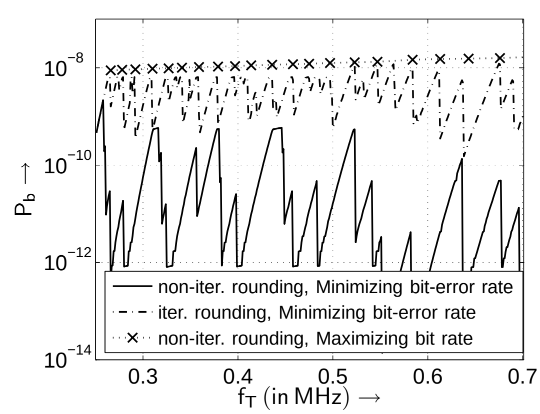

Fig. 6. Aggregate bit-error rates P_{b} as a function of the pulse frequency f_{T} for both power allocation approaches

and their effects were analyzed. In doing so, a transmit power reserve emerges, which on the one hand can be used to increase the bit rate (via a rising pulse frequency). On the other hand the drawback of different subchannel bit-error rates can be avoided using the second approach where the aggregate bit-error rate at a fixed bit rate is minimized. Here the iterative rounding with minimizing the bit-error rate in the second optimization step has lead to the lowest bit rate loss compared to the best results using the bit rate maximization.

For further optimization it might be fruitful to take other setups of bit and power loading into account. We expect, that similar results can be achieved, if they are compared to each other with respect to the optimization criteria considered here.

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References

[1] A. Ahrens and C. Lange, "Power and Bit Allocation for Twisted-Pair Multicarrier Transmission Systems." in *10th International OFDM-Workshop*, Hamburg, August, 31– September, 01 2005, pp. 358–362.

[2] ——, "Power- and bit-loading for twisted-pair multitone systems." in *XXI Krajowe Sympozjum Telekomunikacji (KST)*, Bydgoszcz (Poland), September, 07–09 2005, pp. 63–72.

[3] J. A. C. Bingham, "Multicarrier Modulation for Data Transmission: An Idea Whose Time Has Come," *IEEE Communications Magazine*, vol. 28, no. 5, pp. 5–14, May 1990.

[4] ——, *ADSL, VDSL, and Multicarrier Modulation*. New York: Wiley, 2000.

[5] L. Hanzo, W. T. Webb, and T. Keller, *Single- and Multicarrier Quadrature Amplitude Modulation*, 2nd ed. Chichester, New York: Wiley, 2000.

[6] P. S. Chow, J. M. Cioffi, and J. A. C. Bingham, "A Practical Discrete Multitone Transceiver Loading Algorithm for Data Transmission over Spectrally Shaped Channels," *IEEE Transactions on Communications*, vol. 43, no. 2/3/4, pp. 773–775, February/March/April 1995.

[7] R. F. H. Fischer and J. B. Huber, "A new Loading Algorithm for Discrete Multitone Transmission." in *IEEE Global Telecommunications Conference (GLOBECOM)*, London (UK), November 1996, pp. 724–728.

[8] D. Hughes-Hartogs, "Ensemble Modem Structure for Imperfect Transmission Media," U.S. Patents Nos. 4,679,227 (July 1987), 4,731,816 (March 1988), and 4,833,706 (May 1989).

[9] C. Lange and A. Ahrens, "Orthogonal Bases for the Design of Multicarrier Transmission Systems." in *8th OFDM-Workshop*, Hamburg, September, 24–25 2003, pp. 254–258.

[10] H. S. Malvar, *Signal Processing with Lapped Transforms*. Boston; London: Artech Houser, 1992.

[11] I. Berenguer and I. J. Wassell, "FMT Modulation: Receiver Filter Bank Definition for the Derivation of an Efficient Implementation." in *7th International OFDM-Workshop*, Hamburg, September, 10–11 2002, pp. 158–162.

[12] A. Ahrens and C. Lange, "Equalizer Design for Multicarrier Transmission Systems." in *Nordic HF Conference*, Fårö (Sweden), August, 10–12 2004, pp. 6.3.1–6.3.11.

[13] D. Kreß and M. Krieghoff, "Elementare Approximation und Entzerrung bei der Übertragung von PCM-Signalen über Koaxialkabel," *Nachrichtentechnik Elektronik*, vol. 23, no. 6, pp. 225–227, 1973.

[14] N. Benvenuto, S. Tomasin, and L. Tomba, "Equalization Methods in OFDM and FMT Systems for Broadband Wireless Communications," *IEEE Transactions on Communications*, vol. 50, no. 9, pp. 1413–1418, 2002.

[15] D. Kreß, M. Krieghoff, and W.-R. Gräfe, "Gütekriterien bei der Übertragung digitaler Signale," in *XX. Internationales Wissenschaftliches Kolloquium*. Ilmenau (Germany): Technische Hochschule, 1975, pp. 159–162.

[16] I. Kalet, "The Multitone Channel." *IEEE Transactions on Communications*, vol. 37, no. 2, pp. 119–124, 1989.

[17] J. G. Proakis, *Digital Communications.* Boston: McGraw-Hill, 2000.

[18] D. Daly, C. Heneghan, and A. D. Fagan, "Power- and Bit-loading Algorithms for Multitone Systems." in *3rd International Symposium on Image and Signal Processing and Analysis (ISPA)*, no. 2, Rom (Italy), September 2003, pp. 639–644.

[19] B. S. Krongold, K. Ramchandran, and D. L. Jones, "Computationally Efficient Optimal Power Allocation Algorithms for Multicarrier Communications Systems," *IEEE Transactions on Communications*, vol. 48, no. 1, pp. 23–27, 2000.

[20] L. Goldfeld, V. Lyandres, and D. Wulich, "Minimum BER Power Loading for OFDM in Fading Channel," *IEEE Transactions on Communications*, vol. 50, no. 11, pp. 1729–1733, November 2002.

[21] J. Jang and K. B. Lee, "Transmit Power Adaptation for Multiuser OFDM Systems," *IEEE Journal on Selected Areas in Communications*, vol. 21, no. 2, pp. 171–178, 2003.

[22] J. Jang, K. B. Lee, and Y. H. Lee, "Transmit Power and Bit Allocations for OFDM Systems in a Fading Channel," in *IEEE Global Telecommunications Conference (GLOBECOM)*, vol. 22, no. 1, San Francisco (USA), 2003, pp. 858–862.

[23] C. S. Park and K. B. Lee, "Transmit Power Allocation for BER Performance Improvement in Multicarrier Systems." *IEEE Transactions on Communications*, vol. 52, no. 10, pp. 1658–1663, 2004.

[24] G. D. Forney, R. G. Gallager, G. R. Lang, F. M. Longstaff, and S. U. Qureshi, "Efficient Modulation for Band-Limited Channels," *IEEE Journal on Selected Areas in Communications*, vol. 2, no. 5, pp. 632–647, 1984.

[25] R. M. Corless, G. H. Gonnet, D. E. G. Hare, D. J. Jeffrey, and D. E. Knuth, "On the Lambert W Function," *Advances in Computational Mathematics*, vol. 5, pp. 329–359, 1996.

[26] R. Kohlschmidt and C. Lange, "Ein Beitrag zur Durchsatzerhöhung im Ortskabelbereich," *Frequenz*, vol. 54, no. 5/6, pp. 119–122, May/June 2000.

[27] A. Leke and J. M. Cioffi, "A Maximum Rate Loading Algorithm for Discrete Multitone Modulation Systems." in *IEEE Global Telecommunications Conference (GLOBECOM)*, Phoenix (Arizona), November, 3–8 1997, pp. 1514–1518.